School of Mathematical and Computer Sciences
Actuarial Mathematics & Statistics

Seminars Abstracts, Autumn Term 2003

Leonid Gavrilov and Natalia Gavrilova: Bio-actuarial studies on human longevity

This presentation summaries the results of author's six-year research work at the Center on Aging, University of Chicago, aimed to explore determinants of human lifespan (several projects supported by the National Institute on Aging, USA).

(1) The effects of parental age at person's conception on person's lifespan are studied in a context of mutation theory

(2) The effects of person's month-of-birth on person's lifespan are studied in a context of 'fetal origins of adult disease' concept and the idea of early-life seasonal programming of adult lifespan.

(3) The effects of parental lifespan on person's lifespan are studied in a context of genetics of aging and genetics of quantitative traits.

(4) We also tested widely publicized claims published in Nature (1998), that human longevity comes

with high cost of infertility (half of long-lived women were reported to be childless).

(5) Finally, if time permits, we will discuss possible explanations of aging and longevity in terms of reliability theory.

Iain McPhee: Classification of random walks using Lyapunov functions

I will discuss time-homogeneous random walks on two-dimensional complexes. A two-dimensional complex is a union of a finite number of quarter plane lattices connected at one dimensional boundaries. I will consider the specific case where each boundary belongs to only two quarter planes. All of the results are formulated in a constructive way. By this I mean that for any given random walk we can, with a concrete calculation using the first and second moments of the jumps, conclude whether the process is recurrent or transient. The main new result is for a critical case where the long-term behaviour of the random walk is very similar to that found for walks with zero mean drift inside the quadrants even though this walk has non-zero drifts.

The ideas can be applied directly to two-dimensional exhaustive polling models in critical cases. I will also discuss the non-critical many dimensional model, see S.Foss, G.Last, (Ann.Appl.Probab.,1996). A paper, joint with Mikhail Menshikov, with these results is soon to appear in Ann.Appl.Probab.

Alois Gisler: Multidimensional credibility theory

The following two questions are examples of standard questions which many pricing actuaries are confronted with.

- own data versus industry wide data. Often, own company data as well as industry-wide data are available. But how much should one rely on the own data and how much on industry-wide date when calculating a tariff ?

- "normal" claims and "big" claims In many lines of business, a small number of bigger claims (only 1% or 2% of the total number of claims) make more than half of the total claims load. How should we calculate the pure risk premium corresponding to the big claim part based on rather few observations ?

The appropriate actuarial technique and answer to many of such questions is multi-dimensional credibility. There are two reasons for this:

- credibility is particularly suited in cases with little data

- multidimensional credibility takes simultaneously the observations from the different categories into account and lets the data tell us, whether and how much we can learn from the one category with respect

to the other.

In the seminar, the multidimensional credibility model is presented and the corresponding credibility estimator is derived. Next the methodology is applied to a real data set from motor insurance to estimate the frequency of big claims. At the end, a general result concerning optimal data compression is presented and it is shown, that multidimensional credibility also covers the credibility regression case.

Per Linnemann: Market based valuation of guaranteed benefits of participating life insurance contracts

On 1 January 2002 new life insurance valuation rules were introduced in Denmark. Here the emphasis is on determining a market based liability of the guaranteed benefits of participating life insurance contracts. The paid-up benefit valuation method plays an important role in the new Danish market based life insurance valuation rules. In Linnemann (2000, 2002, 2003a) we present the theoretical background to the paid-up benefit valuation method for level premium paying participating life insurance contracts. Moreover in Linnemann (2002) we give a theoretical basis for amendments and further developments of the new Danish market based life insurance valuation rules. We suggest that the so-called extended paid-up benefit valuation method should be used for the valuation of the guaranteed benefits of participating life insurance contracts. The cash-flows that should enter in the calculation of a market based value of the guaranteed benefits of participating life insurance contracts are being determined by this method. We point out that the valuation principles of `coherence between the benefits and premiums being valued' and `avoidance of future valuation strains' are relevant in a market based valuation regime. In Linnemann (2003b) we review the above papers.

We present in the lecture a number of the principal considerations and results that have been dealt with in the above papers.

Linnemann, P. (2000). An actuarial analysis of participating life insurance. Pen-Sam, Working Paper, August 2000. Published in Scandinavian Actuarial Journal 2003, 153-176.

Linnemann, P. (2002). Valuation of participating life insurance liabilities. Pen-Sam, Working Paper, April 2002. To be published in Scandinavian Actuarial Journal.

Linnemann, P. (2003a). An actuarial analysis of participating life insurance. Scandinavian Actuarial Journal 2003, 153-176. Errata 177.

Linnemann, P. (2003b). Market based valuation of guaranteed benefits of participating life insurance contracts. Pen-Sam, Working Paper, June 2003.

Hans Buhlmann: Valuation portfolio and risk management

Actuarial valuation should be understood in a multidimensional sense. In practice this means that the actuary should express the liabilities of the insurer as a portfolio of financial instruments. This Valuation Portfolio can be calculated policy-wise. For risk management purposes the aggregated valuation portfolio has to be compared with the investment portfolio.

Zbigniew Palmowski: Markov processes conditioned to never exit a substate space with application to a single server queue

In this talk we consider a continuous time Markov process killed at the exit time T from a subset A of its state space. We assume that T is finite a.s. We define the concept of Never Exiting (NE) Markov process from A.

The first question is to determine when this process exists and the second is whether we can define it by the change of probability measure argument.

It turns out that NE process is Markovian and we will study its properties. In particular we give the relationship between the stationary distribution of NE Markov process and the quasi-stationary distribution. The general scheme can be found in the papers Jacka and Roberts (1995) and Lambert (2000). We apply the results to a workload process of G/G/1 queue conditioned to stay positive. We consider the cases when the service times distribution is light-tailed and regularly varying.

Andreas Kyprianou: Law of the Iterated logarithm for oscillating random walks conditioned to stay positive

An oscillating random walk (whose increments have second moments) when conditioned to stay positive may be seen in some sense as an analogue to a Bessel-3 process; since a Bessel-3 is also equal in law to a Brownian motion conditioned to stay positive. Like Brownian motions, Bessel-3 processes obey LILs at large times. It is therefore natural to ask if, like random walks with second moments, an oscillating random walk conditioned to stay positive also obeys an LIL at large times.

Using three fundamental facts: 1) the Bertoin-Doney description of the step distribution of conditioned random walks, 2) Tanaka's fundamental path decomposition of conditioned random walks and 3) a new Skorohod-type embedding of conditioned random walks in Bessel-3 processes, we establish an LIL result as well as LIL-type results for the slowest growth rates.

This is joint work with G. Kersting (Frankfurt) and Ben Hambly (Oxford).

Denis Denisov : The maximum on a random time interval of a random walk with heavy-tailed increments and

infinite mean

Consider a random walk $S_n=\xi_1+...+\xi_n$ with i.i.d. increments. S. Asmussen has found the exact asymptotics for tha tail distribution of the maximum $M_\tau=\max_\{0\le i \le \tau\} S_i$ on a random time interval where $\tau=\min\{n\ge 1: S_n\le 0\}$. He assumed the condition $-\infty< E\xi_1<0$ to hold. We give a complementary result in the case $E|\xi_1|=\infty$.

Seva Shneer : Estimates for the tail distributions of sums of subexponential random variables

Let $\{\xi_i\}$ be a sequence of i.i.d. random variables and $S_n = \sum_{i=1}^n \xi_i$. For two classes of subexponential distributions, we obtain new uniform upper bounds for the ratios $P(S_n > x) / P(\xi_1 > x)$. Then we apply these bounds to the asymptotic study of a Markov-modulated random walk with heavy-tailed increments.

Mark Willder: Management of a With-Profit Fund using Option Pricing Techniques

Traditionally the cost of the guarantees under UK unitised with-profit policies has been ignored or priced in a very imprecise way. However in this paper I will charge the policyholder for these guarantees an amount equal to the price of matching put options. Using simulations I show how the distribution of payouts compares under different levels of guarantees. Further I investigate the size of the free estate if charges are deducted according to option prices but the estate is actually invested in bonds.

Laurent Massoulie: Random graph models of peer-to-peer systems

The focus of this talk is on peer-to-peer systems, and more precisely on what peer relations to maintain in such systems. Those peer relations are naturally modelled as a graph, and one question of interest is how to create or adapt such a graph so as to meet desired reliability objectives, e.g. that connectivity be retained with a given proportion of random link failures. One constraint specific to the context of peer-to-peer systems is that graph adaptation should involve distributed / local operations only.

I will describe local rules for adapting a given graph so as to improve its reliability, and an analysis of the resulting graph's connectivity properties. If time allows I will also discuss a somewhat related issue, namely how to sample uniformly from the node set of a graph.

Mark Owen: The Super Replication Price of an Unbounded Contingent Claim: Utility Induced Restrictions on Negative

Wealth

It is well known that in an incomplete financial market, the super replication price of a contingent claim coincides with the supremum of its expected values over the set of pricing measures. For the case of a contingent claim which involves possibly unbounded losses however, super replication using only admissible trading strategies would lead to a gap between the interval of prices and the super replication price - the choice of permissible strategies becomes crucial.

Consider a financial market in which an agent is permitted to trade with only utility-induced restrictions on negative wealth. For a sufficiently integrable (but possibly unbounded) contingent claim, we give a representation of the utility-based super-replication price of the claim as the supremum of its discounted expectations under pricing measures with finite generalised entropy.

Central to the proof of this result is a bipolar relation between the cone of super replicable contingent claims with zero initial endowment, and the cone generated by pricing measures with finite loss-entropy.

Julia Wirch: Iterated CTE with applications to Equity Linked Guarantees

A method is presented for defining a dynamic risk measure from a static risk measure using backwards iteration. This method is applied to the CTE risk measure to produce the iterated-CTE (ICTE). It is shown that the ICTE is coherent, consistent and relevant. Formulae for the ICTE when the loss process is lognormal are shown. Implementation of the ICTE to equity-linked insurance with maturity and death benefit guarantees is discussed.

Steve Buckland: Fitting stochastic population dynamics models to spatio-temporal data

When modelling change in animal abundance, empirical modellers typically ignore the population processes. Hence they can obtain estimated rates of change that are biologically implausible, and they have no mechanism for predicting the effects of different management strategies, or for modelling movement between components of a metapopulation. Conversely, mathematical modellers have typically failed to integrate fully stochasticity in the population processes, and uncertainty in estimates of population parameters. In this talk, we show how state-space models provide a framework for embedding stochastic population dynamics models fully into inference. A framework is also presented that allows complex models to be constructed as a sequence of simple process models.

Seminar Timetable

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