New approaches to study historical evolution of mortality (with implications for forecasting)

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Abstract.   Increasing numbers of people surviving to advanced ages pose serious challenge to government pension systems and to most developed societies. Therefore, accurate estimates of mortality at advanced ages are essential to improving forecasts of mortality and the population size of the oldest old age group. In this paper we present some new approaches to mortality and population projections at older ages. We apply modified method of mortality shifting to the data of Sweden and make mortality projections up to year 2050. Specifically, we identify the best historic time interval for identifying the rate of mortality decline to use in mortality extrapolation. In the case of Sweden, the best historic interval is 1980 through 2008 years for both men and women. For men, the rate of mortality decline is almost twice as high as this rate for women. Using assumptions about log-linear decline of mortality over time and the exponential increase of mortality with age we conducted mortality projections for Swedish population over the next 50 years. According to these projections, life expectancy at age 25 will increase from 54.07 years in 2005 to 62.71 years in 2050 for men and from 58.20 years to 63.50 years for women. These advances in life expectancy will not result in the growth of native population and it is expected that the native population of Sweden starts to decline after 2036 (assuming the birth rate remains unchanged).

1 Introduction

Population aging is a global phenomenon, which is particularly expressed in industrialized countries. The proportion of older people in these countries grows with accelerated pace mainly due to increasing longevity, because fertility there already reached very low levels. Population aging is expected to continue over the next few decades, eventually leading to the global convergence in the proportion of older people. Although fertility decline was the main cause of population aging in the past, the process of population aging in industrialized societies is determined by declining mortality at older ages.

Thus, mortality studies and projections for older ages are particularly important for making accurate demographic forecasts of population aging.

This study is focused on mortality changes at older ages because these changes are now the main driving force behind both increases in life expectancy and population aging. In this study we present some new approaches to mortality forecasts and population projections at older ages.

2   Using parametric models (mortality laws) for mortality projections

Parametric models of mortality represent a useful tool in demographic and actuarial projections of mortality. One of the first and most successful attempts to express the dependency between mortality and age mathematically was that of the English actuary Benjamin Gompertz, dating to 1825 (Gompertz[13]):

$$\mu_x = R_0 \exp(\alpha x)$$

where μ_x is the force of mortality at age x; and α and R_0 are the parameters of the equation. This formula, which describes the mortality of people older than 20, was called the Gompertz law, and its parameters were named the Gompertz parameters. Subsequently, the Gompertz law began to be used widely for describing mortality of laboratory animals (Gavrilov and Gavrilova[6]).

In his work, Gompertz noted that, in addition to mortality, which grows exponentially with age, there can also exist a component of mortality which is independent of age: "It is possible that death may be the consequence of two generally coexisting causes: the one chance, without previous disposition to death or deterioration, or increased inability to withstand destruction" (Gompertz[13]). However, for the analysis of the life tables which were then available, Gompertz considered it possible to restrict himself solely to the exponential component of mortality. Not until 35 years later, in 1860, did another actuary William Makeham add the age-independent component to the Gompertz formula (Makeham[19]). This component, usually denoted by the letter A, received the name of the Makeham parameter (Gavrilov and Gavrilova[6]). Thus the formula appeared which we now know as the Gompertz-Makeham law:

$$\mu_x = A + R_0 \exp(\alpha x)$$

A is the age-independent component of mortality, which we called the background component of mortality in analogy with background radiation (Gavrilov and Gavrilova[8]; Gavrilov and Gavrilova[6]). The second term of this equation is the age-dependent component of mortality (Gavrilov and Gavrilova[8]), which is called now the senescent mortality (Bongaarts[1]). As can be seen from what has been said, the age-dependent component of mortality is an exponential. In particular case in which the background mortality can be

ignored (for example under good laboratory conditions or in contemporary industrialized countries), the total force of mortality grows exponentially with age, i.e. in accordance with the Gompertz law.

The Gompertz-Makeham formula describes life span distributions of a wide variety of biological species (drosophila, mosquitoes, flour beetles, mice, rats, horses and mountain sheep), including humans (Gavrilov and Gavrilova[6]). There are some reports that the competing Weibull formula (power law) fits data better than the Gompertz formula. These reports are usually based on the analysis of a few life tables for populations of small size (often less than 100 animals). Our comparative study of Weibull and Gompertz models using data for 260 human life tables and 15 life tables for fruit flies (with initial population size of 1000 or more animals) demonstrated that on average the Gompertz law fits mortality at adult ages better than the Weibull law (Gavrilov and Gavrilova[6]).

Subsequently, there were many attempts to modify the Gompertz law. The most common way of modifying the Gompertz function is to use what are called logistic equations. The earliest such formula was proposed by Perks and the latest and the most widely used one was proposed by Kannisto and is called a Kannisto formula (Kannisto[16]):

$$\mu_x = \frac{B\exp(\alpha x)}{1 + \text{B}\exp(\alpha x)}$$

The formulas listed above are applicable to mortality of adult population (usually above age 20 years). There were also attempts to describe mortality in the entire age interval, such as Heligman-Pollard (Heligman and Pollard[14]) and Siler (Siler[21]) formulas.

Parametric formulas can be used in population projections by analyzing historical trends of their parameters. For example, in 1979, during analysis of historical changes in mortality of the Swedish male population, it was found that the age-dependent component of mortality in the Gompertz-Makeham formula demonstrates surprising historic stability, in contrast to rapid decline in age-independent mortality (Makeham term) (Gavrilov and Gavrilova[8]). Further more careful investigation confirmed the validity of this phenomenon (Gavrilov *et al*.[10]; Gavrilov and Gavrilova[6]) and the study of historical time series of mortality for 17 countries permitted the conclusion that it was quite general in character (Gavrilov and Gavrilova[6]). Figure 1 shows changes in total, background and senescent mortality for Swedish males. It can be seen that the background component of mortality is the only mortality component, which has significantly changed over the period under investigation (1900-1970). The senescent mortality (and two Gompertz parameters), turn out to be practically unchanged, despite the sharp fall in total mortality in the 20th century. We observe that the substantial decline in mortality rates in Sweden at the beginning of the 20th century can be explained by a decrease in the Makeham component, while the Gompertz component remained virtually

constant during the same period. In the 1960s, as the Makeham component had almost reached zero, it became foreseeable that the rapid decline in mortality rates would come to an end. And this is what in fact happened in the 1960s (see figure 1).
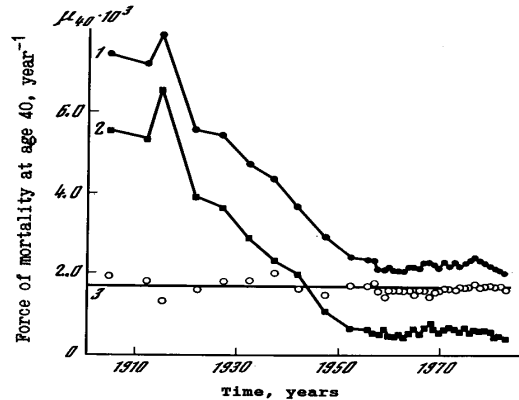

Fig. 1. Historical changes of age-independent (background) and age-dependent (senescent) mortality for Swedish males. Source: Gavrilov *et al.*[10].

Thus, based on the observation of the mortality tables for the first half of the 20th century, it was possible to predict an apparent "biological limit" to the force of mortality. To test this approach, we studied a number of other countries (Gavrilov and Gavrilova[6]). This phenomenon was confirmed in the study of 17 countries during the period of 1900-1970 years. For example, at the beginning of the 20th century, total mortality was substantially higher in Norway than in Denmark. However, based on the observation that the Gompertz component of mortality was considerably lower in Norway, we were able to predict a reversal in the mortality differences, as the Makeham component declines. This is exactly what happened. In Italy, the mortality rates of men and women were virtually identical at the beginning of the 20th century but the biological limit for women was lower due to their lower Gompertz component. Based on data from the beginning of the 20th century, we were able to predict that eventually the female mortality would become lower than male mortality. These examples could be continued (Gavrilov and Gavrilova[6]). However after the 1960s, a new unexpected trends in mortality have started, which are discussed now. These trends were not well visible at the time of our earlier study, although some indications of further mortality decline have been already noticed (Gavrilov and Nosov[11]; Gavrilov and Gavrilova[6]).

Recently Bongaarts developed further the method based on studying historical trends of Gompertz-Makeham parameters, suggesting use of logistic formula for mortality forecasting. This modification was reasonable because mortality rates

for period life tables in Human Mortality Database used in his study were fitted by logistic formula after age of 85 years (Wilmoth *et al.*[23]). His study analyzed historical trends in the interval 1950-2000 years for 14 countries and confirmed decline of the background mortality and stability of the slope parameter in the Gompertz term found in our earlier studies. Due to the limited number of life tables (no life tables before 1950 were used) this study could not demonstrate the full scale of decline in background mortality during the first half of the 20th century. However this study revealed another interesting regularity that could not be fully analyzed in the past: decline in the pre-exponential multiplier of senescent mortality (intercept coefficient of the Gompertz function). Decline of this parameter (called the level parameter by Bongaarts) in conjunction with stability of the slope parameter in the Gompertz term means that the senescent component of mortality in developed countries undergoes parallel shift in semi-log coordinates over time. This pattern of mortality change was called a shifting logistic model (Bongaarts[1]). Based on this mortality pattern, Bongaarts suggested a new approach to mortality projections. This approach is based on estimating parameters of the logistic formula for a number of years and extrapolating the values of three parameters (background mortality and two parameters of senescent mortality) to the future (Bongaarts[1]; Bongaarts[2]).

Figure 2 illustrates the two phases of mortality decline in the 20th century. When we analyze mortality data on a larger time scale, we observe a decline of mortality in all age groups between years 1925 and 1955, except for the elderly where death rates remained relatively constant. Note that mortality age trajectories for years 1925 and 1955 are close to each other at older ages, which corresponds to the stability of the senescent mortality during the first half of the 20th century. After the 1950s, the parallel shift of mortality has been observed, which corresponds to the model proposed by Bongaarts (for example, compare mortality trajectories for 1955 and 2005).

It should be noted that in addition to the approach based on the background and senescent mortality components there is another way of mortality partitioning. In 1952, Jean Bourgeois-Pichat attempted to predict population mortality using the idea of endogenous and exogenous causes of death (Bourgeois-Pichat[3]). In the exogenous causes of death he included infectious and parasitic diseases, respiratory diseases, accidents, poisonings, and violence. The endogenous causes of death included malignant neoplasms, circulatory diseases, and the remaining causes of death (Bourgeois-Pichat[3]).   In Bourgeois-Pichat's opinion, the evolution of human mortality can be likened "to the erosion of soil composed of two kinds of rock: soft rock and hard rock." At first, the "soft rock" is quickly eroded (exogenous causes of death), then the "hard rock" slowly erodes (endogenous causes of death). On the basis of these ideas, it was predicted that medical advances in eliminating the exogenous causes of death would lead to the endogenous causes of death coming to the fore. Thus, Bourgeois-Pichat formulated "the concept of a temporary limit on mortality

decline", and even calculated the level of this "temporary limit" for each age, calling it "the biological limit of mortality decline" (Bourgeois-Pichat[3]; Bourgeois-Pichat[4]).
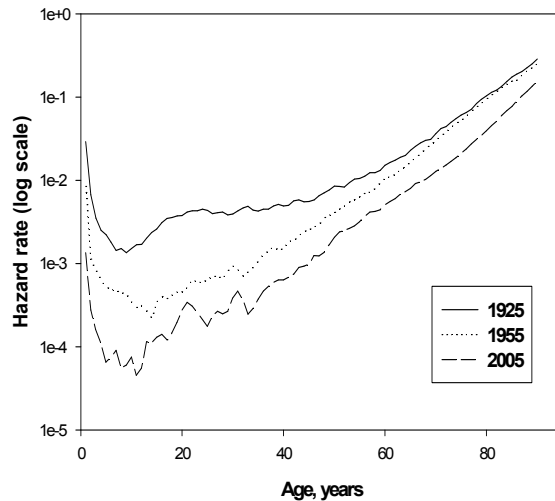

Fig.2. Changing patterns of mortality decline among Swedish females.

Later Carnes and Olshansky developed this approach further suggesting so-called biologically motivated partitioning of mortality into extrinsic and intrinsic mortality based on cause-of-death information (Carnes and Olshansky[5]). Information on causes of death is used sometimes in actuarial practice for making mortality projections, although these projections usually underestimate future mortality decline. Mortality partitioning proved to be a useful tool for mortality projections in the past when background mortality (analog of extrinsic mortality) was high, but this approach is less useful now when background mortality is close to zero and does not change over time. In addition to that, in some cases, it is simply impossible to establish whether death is exogenous or endogenous. For example, a patient may be suffering from several diseases, none of which alone would lead to death, but which are lethal in combination. So this approach has a limited applicability in demographic practice now.

Despite the usefulness of parametric approach to mortality projections it has serious limitations. The main limitation is a dependence on the particular formula, which makes this approach too rigid to respond to possible changes in mortality trends and fluctuations. In the next section we consider some methods of mortality projections based on non-parametric approaches.

3 Nonparametric approach to mortality projections

The Lee-Carter method is now one of the most widely used methods of mortality projections in demography and actuarial science (Lee and Carter[18]; Lee and Miller[17]). Its success is stemmed from the shifting model of mortality decline observed for industrialized countries during the last 30-50 years. The Lee-Carter method is applied to the logarithm of mortality and is based on the following formula for hazard rate (or central death rate):

$$\ln(\mu_{x,t}) = a(x) + b(x)k(t)$$

Note that Lee-Carter model is based on multiplicative model of mortality change over time (rather than additive one as in the case of Gompertz-Makeham model).   However the Lee-Carter model is not based on any particular parametric formula and models each age group separately. The Lee-Carter model allows researchers to make a compact description of a large set of mortality data without excessive loss of information. In contrast to aggregated indicators such as life expectancy, the knowledge of the Lee-Carter model parameters allows researchers to reconstruct values of age-specific mortality rates and their temporal evolution with reasonable accuracy. One limitation of this method is related to the assumption of Lee-Carter model, that historical evolution of mortality at all age groups is driven by one factor (parameter) only. However a factor analysis of mortality evolution found that this approach turns out to be overly simplistic (Gavrilov and Gavrilova[6]; Gavrilov and Nosov[11]). For example, factor analysis of mortality dynamics over the period of 1900-2007 in developed countries found that at least two time-dependent factors are responsible for observed decline of mortality (younger age groups have a different factor of mortality decline compared to older groups). One-factor model was applicable to earlier historical periods only (before the 1950s), when a decline in mortality rates was driven mainly by a decrease of the background mortality (the Makeham parameter of the Gompertz-Makeham law) (Gavrilov and Gavrilova[6]; Gavrilov *et al.*[10]). It is obvious that the Lee-Carter model is not well applicable to mortality modeling during the period 1900-1950 because of additive rather than multiplicative pattern of mortality decline during this time.

In order to overcome limitation of one-factor model of mortality and to determine the true number of factors underlying mortality changes over time, we conducted a factor analysis of mortality for Swedish data over the period of 1900-2008. We used the so-called P-technique of factor analysis when the analysis occurs across different time points or observations (values of hazard rates at different years) for ages 25 through 85 years (Überla[22]). We applied factor analysis procedure with promax rotation method using the Stata, release 11 statistical package. Data on males and females were analyzed separately. We identified two factors capable of explaining almost 99% of the variance in

the temporal changes of hazard rates. Thus, for more accurate description of mortality evolution, the following model would be preferable:

Mortality force (age, time) = a_0(age) + a_1(age) x F_1(time) + a_2(age) x F_2(time)

where a_0(age), a_1(age), a_2(age) are three parameters depending on age only, while F_1(time) and F_2(time) are two parameters depending on time only (sets of coefficients determined by factor analysis models).

By studying the variation of these factors over time, we noted that the first factor - comparable to the Makeham component and observed in the "young ages" population - declined from the beginning of the century. The second factor - comparable to the senescent mortality and chiefly concerning the "old ages" population - remained remarkably stable over a period of 1900-1950 (see Figurers 3 and 4). Without more recent data, we might predict continued historical stability of this factor. However, a radical change has occurred after the 1950s and mortality has begun to decline among older people while the mortality of the younger age groups has already reached a level close to zero. Thus, factor analysis of the time series of mortality confirms the preferential reduction in the mortality of old-aged people in the recent years. Also note that for males, the senescent factor started its rapid decline significantly later compared to females.
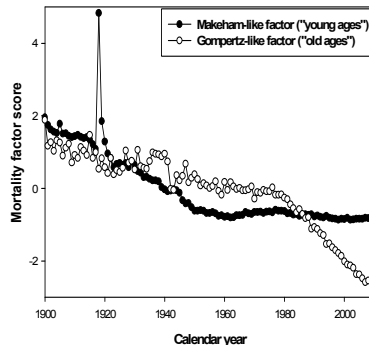

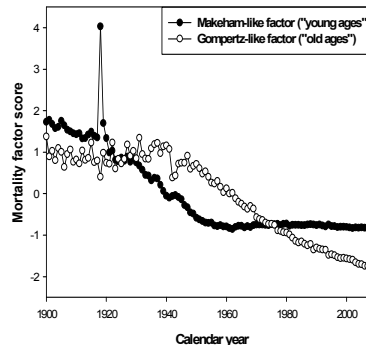**Fig. 3**. Time dependence of factor scores for "young-age" and "old-age" factors for Swedish males.

Fig. 4. Time dependence of factor scores for "young-age" and "old-age" factors for Swedish females.

Observations made before 1950 enrich the historical data as a whole but they are liable to distort the results of mortality projections. However, for the future forecasts it is better to use more recent data, which take account of the change in the patterns of mortality decline. What conclusions may be drawn at this point?

In the past, it was possible to argue that there was a biological limit underlying the observed mortality rate. We have observed, however, that these limits can be pushed back thanks to technological or medical progress. Although it is equally impossible to conclude that the mortality force is tending toward zero, the short-term trend is clearly oriented downwards.

The approach based on the factor analysis has several advantages. First it is able to determine the number of factors affecting mortality changes over time. Second, this approach allows researchers to determine the time interval, in which underlying factors remain stable or undergo rapid changes. For example, Figures 3 and 4 clearly demonstrate that the second factor was relatively stable in the past but now it is rapidly declining for both men and women. However, this rapid decline started almost 30 years later in men compared to women. Most methods of mortality projections are not able to identify the best time interval for mortality changes that should be extrapolated in the future. For example, the Lee-Carter method suggests using the longest possible interval. It is clear that such approach will not bring the most accurate mortality forecast. Using the results of factor analysis we may conclude that 1980 is the best starting year for Swedish mortality projections. After this year the senescent factor demonstrates a stable almost linear decline and it is reasonable to suggest that this decline will continue into the foreseeable future.

Taking into account the shifting model of mortality change it is reasonable to conclude that mortality after 1980 can be modeled by the following log-linear model with similar slope for all adult age groups:
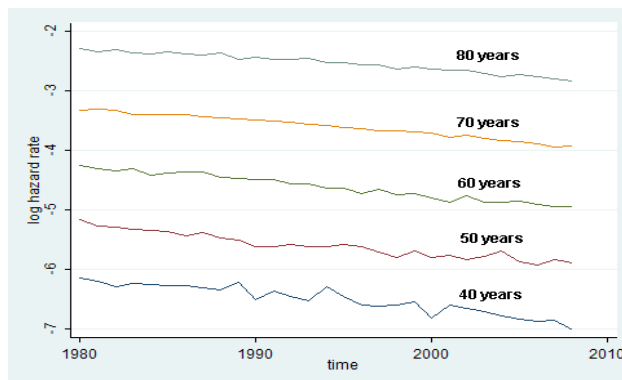
$$\ln(\mu_x) = a(x) + b(x)k$$

Fig. 5. Linear changes of logarithm of hazard rate at different ages after 1980 for Swedish males.

Figure 5 illustrates the validity of the suggested model for Swedish men. Note that the logarithm of mortality declines linearly in all observed age groups with the same linear slope. Similar regularity is observed for Swedish women.

We suggest here to use the shifting model of mortality with the slope parameter of historical decline in mortality, based on rate of mortality change after 1980. We analyzed mortality in the age interval 25-85 years. However, there is a question related to the mortality pattern at advanced ages. Bongaarts used logistic formula for mortality modeling and this formula is now the most popular way of mortality modeling at advanced ages. Should we use this formula for mortality projections at advanced ages? Our study showed that the Gompertz formula fits mortality up to age 106 years better than the logistic (Kannisto) formula (Gavrilov *et al.*[9]).

4 Making mortality forecasts

In our previous study we demonstrated that the traditional Gompertz model can be successfully used for mortality modeling up to very old ages (Gavrilov *et al.*[9]). Extending this model to age 106 years is sufficient for most countries to close life table because only few individuals survive to these ages even in the countries with low mortality. The factor analysis of mortality changes indicates that age-dependent (senescent) mortality continues to decline over time, and this decline does not demonstrate any indications of slowing down. This observation means that the traditional Lee-Carter approach can be used for mortality forecasting. The study of mortality changes using factor analysis shows that mortality trends after year 1980 can be continued beyond 2011 for both men and women. Figure 5 shows mortality trends after 1980 for Swedish males. Note that logarithms of mortality demonstrate practically linear decline with time with similar slope for the studied ages. We can use this property to model mortality decline after 2010 assuming the same rate of mortality changes (in a log scale) for different ages. Changes of mortality after 1980 for Swedish females reveals a similar phenomenon, although women demonstrated slower rate of mortality decline.

In the suggested mortality projections we assumed that mortality of Swedish males declines by 2 percent per year in all age groups, while mortality of Swedish female declines by 1 percent per year in all age groups. This gender difference in the rate of mortality decline over the last three decades is apparently responsible for the decreasing gender gap in life expectancy observed in the majority of developed countries (Glei and Horiuchi[12]).

Additionally, we assume that mortality follows the Gompertz law at advanced ages as discussed in the previous section. Also, we do not attempt to close life table at any predetermined age (say, 110 years) as it is often made in demographic forecasts. In our projections, the number of the last death is shifting to higher ages as long as mortality continues to decline. Using all the listed assumptions we conducted mortality projections for the next 50-60 years.

Our projections of life expectancy using this approach are more optimistic than forecasts of the majority of demographers made so far. According to our forecasts life expectancy at birth may reach 90 years in 2070. Another difference from the existing projections is the declining gender gap in life expectancy. It is projected that in 2059 life expectancy of men may outrun that of women if current trends in mortality continue to the future. Figure 6 shows the projected trends in life expectancy at age 25 for Swedish men and women together with the observed values of life expectancy taken from the Human Mortality Database. Note that our method shows good correspondence with real data for male mortality (the first three calendar years) while for females it overstates the observed life expectancy. Overestimation of life expectancy for women suggests that the selected pace of mortality decline (1 percent per year) is too high for age groups not considered in our analyses (probably age groups over 90). Although we made mortality projections up to year 2070, it is more reasonable to suggest that the observed trend in mortality decline will continue for the next decade with possible uncertain changes after 2020. In this case, the projected life expectancy at birth in 2020 will be 81.53 years for men and 85.16 years for women with possibility of lower value of life expectancy for women. Thus, gender gap in life expectancy will decrease from 4 years in 2006 to 3.5 years in 2020.
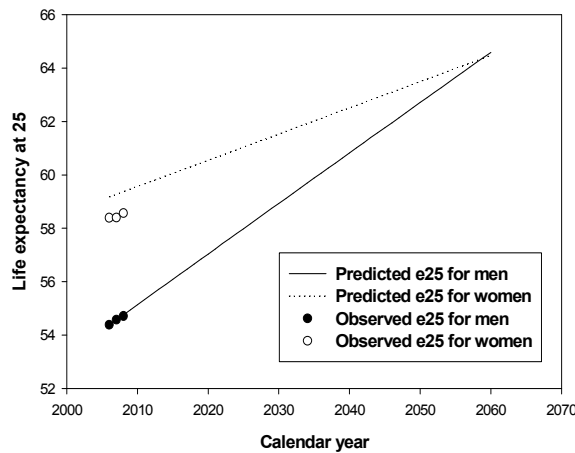

Fig. 6. Projected trends of adult life expectancy (at 25 years) for Swedish men and women.

These changes in life expectancy will have a profound effect on population aging. Increasing longevity accelerates the pace of population aging. We made computer model projections for growth of the proportion of older persons aged 65 and older in the Swedish population and found that in the nearest future Sweden will experience very rapid population aging, so that the projected

number of persons aged 65 and older will reach 25% for men and 28% for women (currently 17 and 21 percent). Current differences in the degree of population aging among men and women will become significantly smaller by 2055.

Conclusions

We demonstrated that using factor analysis and simple assumptions about mortality changes allowed us to provide simple and realistic mortality forecasts. It is obvious that this approach should be country-specific, because each country may demonstrate its own pattern and factor structure of mortality decline over time. However, our preliminary analyses show that the two-factor pattern of mortality decline is observed for the majority of industrialized countries. This study assumes that no changes in mortality patterns are expected in the future. This is most likely an overly simplistic view. Old-age mortality may be affected by different tendencies in the future. On the one hand, an anticipated longevity revolution and new anti-aging technologies are able to slow down the aging process resulting in significant decline of mortality at older ages (Illes *et al*. [15]). On the other hand, epidemics of obesity and diabetes in developed countries may slow down future mortality decline (Olshansky *et al*.[20]). These multidirectional trends will shape the pattern of mortality changes in the coming decades and affect population aging in industrialized countries. One important conclusion comes from the mortality projections presented here: these profound future declines of mortality will not result in overpopulation. Without migration and fertility changes, the native population of Sweden will undergo depopulation after 2025. Moreover, it was shown that population changes are surprisingly slow in their response to a dramatic life extension. For example, we applied the cohort-component method of population projections to 2005 Swedish population for several scenarios of life extension and a fertility schedule observed in 2005. Even for very long 100-year projection horizon, with the most radical life extension scenario (assuming no senescence after age 60), the total population increases by 22% only (Gavrilov and Gavrilova[7]). Thus, the future life extension will not significantly increase the total population number, although it significantly accelerates the future population aging.

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